

# Theoretical Study on Electroelastic Fracture Mechanics of Piezoelectric Material Systems

著者	林 森
号	50
学位授与番号	3603
URL	<a href="http://hdl.handle.net/10097/37271">http://hdl.handle.net/10097/37271</a>

氏 名	りん しん 林 森
授 与 学 位	博士 (工学)
学 位 授 与 年 月 日	平成18年3月24日
学位授与の根拠法規	学位規則第4条第1項
研究科, 専攻の名称	東北大学大学院工学研究科 (博士課程) 材料加工プロセス学専攻
学 位 論 文 題 目	Theoretical Study on Electroelastic Fracture Mechanics of Piezoelectric Material Systems (圧電材料システムの電気弾性破壊力学に関する理論的研究)
指 導 教 員	東北大学教授 進藤 裕英
論 文 審 査 委 員	主査 東北大学教授 進藤 裕英      東北大学教授 山中 一司 東北大学教授 裘 進浩

## 論 文 内 容 要 旨

### Chapter 1 Introduction

Piezoelectric ceramics and composites are widely used in sensors and actuators in the field of smart materials and structures due to their prominent electromechanical coupling features. However, piezoelectric ceramics and composites are brittle and susceptible to fracture. Electroelastic field concentrations at defects or inhomogeneities such as cracks, voids, particles or electrodes in piezoelectric ceramics and composites can contribute to critical crack growth and subsequent mechanical failure or dielectric breakdown. Therefore, understanding of the fracture behavior in the piezoelectric ceramics and composites would provide information for tailoring mechanically reliable piezoelectric material systems and to improve the design of electromechanical devices as they find more applications.

The analysis of crack problems in piezoelectric materials is complicated in comparison to the elastic materials because of the anisotropic electro-mechanical properties and the electromechanically coupling effect. In the theoretical studies of the piezoelectric crack problems, the researchers have different opinions about the boundary conditions at the crack surfaces. Usually, as the dielectric constant of the air or the medium between the crack faces is very small as compared to that of the piezoelectric material, some researchers have assumed crack surfaces to be free of surface charge (the so-called condition of impermeability). On the other hand, some researchers have assumed that the normal component of the electrical displacement and tangential component of the electric field are continuous across the crack face (the permeable crack boundary condition).

In the study of piezoelectric failure, available solutions in pure elastic materials have been extended to the corresponding problems in piezoelectric materials. The classical theory of fracture mechanics is concerned only with stress intensity factor  $K$ , which merely indicates the ratio of the elevation of the local stress to that of the applied stress. However, the classical single parameter  $K$  theory is not adequate for the piezoelectric crack problems since the so-called "stress intensity factor" itself is not a criterion of piezoelectric failure. The energy release rate  $G$  and energy density factor  $S$  have been found to be the most useful and realistic piezoelectric failure criteria.

Recently, a finite element analysis was made for the single-edge precracked piezoelectric ceramics for various electric fields to calculate the total potential energy release rate and mechanical strain energy release rate for permeable and impermeable crack models, and the single-edge precracked-beam (SEPB) tests were performed on lead zirconate titanate (PZT) ceramics to verify theoretical predictions of the influence of the applied electric field on the piezoelectric fracture behavior. It is concluded that the impermeable boundary conditions should not be used in engineering practice because the criteria are unreliable and may yield misleading results. Theoretical analyses on cracked piezoelectric ceramics also indicated that a negative energy release rate is produced for the impermeable crack model. The finite element analyses of the SEPB and indentation fracture (IF) tests were performed on PZTs based on the use of the permeable and impermeable boundary conditions. It is found that for a given residual force derived from the indentation plastic zone, the positive electric fields decrease the energy release rate at maximum depth point of the permeable crack, while negative electric fields have an opposite effect. The increase in the crack length with increasing positive electric field is attributed to the increase of energy release rate with increasing positive electric field.

All of these analytical and experimental results directly highlight the need for a more thorough understanding of piezoelectric crack behavior. It has been shown that the complex combination of loading conditions (mechanical loading and applied electric fields), crack geometry and material properties, strongly influence the crack behavior of piezoelectric material system. Also, the nonlinear effect caused by localized polarization switching may affect the piezoelectric crack behavior.

Fracture mechanics has been applied throughout the world for the design of any devices and structures where sudden, catastrophic failure would cause serious consequences. The stress intensity factor, energy release rate and energy density factor could be used as extremely effective correlation parameters to brittle fracture. Theoretical methods are essential for solving crack problems for two main reasons. First, they provide the correct form of singularities and asymptotic results that may be needed to analyze and interpret the experimental results and to use for improving the accuracy of purely numerical solutions. Secondly, they provide accurate solutions for relatively simple crack geometries and for certain idealized material behavior that could be used as benchmarks for numerical and approximate procedures. The finite element method appears to be ideal for solving crack problems in the devices and structures and is widely used in fracture mechanics.

The purpose of this paper is to study the effects of mechanical loading (applied stress and applied strain), electric fields, crack and material geometries, and electroelastic properties on the two- and three-dimensional crack behavior of piezoelectric ceramics and composites. The influence of crack face electrical boundary conditions and localized polarization switching near the crack tip on the piezoelectric crack behavior is also discussed.

## **Chapter 2 Electroelastic fracture mechanics analysis of piezoelectric ceramic strip**

The effects of crack face boundary conditions on the piezoelectric fracture mechanics are discussed by analyzing the plane strain electroelastic problem of an orthotropic piezoelectric ceramic strip with a central permeable or impermeable crack. The problem of a long strip is formulated by means of integral transforms and reduced to the solution of a system of Fredholm integral equations of the second kind. Fracture mechanics

parameters such as stress intensity factor, energy release rate and energy density factor based on both permeable model and impermeable model are compared. A finite element method is also used to calculate these fracture mechanics parameters, and the results are compared with the exact solutions. The numerical results illustrate that the impermeable assumption can lead to significant errors regarding the effects of the electric fields on crack propagation.

### **Chapter 3 Comparison of energy release rate and energy density criteria for a piezoelectric layered composite with a crack normal to interface**

A critical comparison of the energy release rate and energy density criteria is made using the example of a piezoelectric layer bonded between two half-spaces of a different elastic solid containing a crack normal to the interfaces. Numerical values of stress intensity factor, energy release rate and energy density factor are presented to exhibit electroelastic interactions. Considered are the exact (permeable) and impermeable crack models. The energy release rate criterion for the impermeable crack model led to negative values which are unphysical. This is consistent with previously published results that seem to contradict with experimental observation related to crack growth enhancement and retardation. The energy density factor always remains positive. This shows that a knowledge of the stress intensity factors alone is not sufficient to explain the behavior of fracture in piezoelectric materials.

### **Chapter 4 Electroelastic fracture mechanics analysis of central active piezoelectric transformer**

The theory of linear piezoelectricity is applied to solve the plane strain electroelastic problem of a central active piezoelectric transformer in presence of a crack located normal to the interfaces. The ends of the crack are situated at equal distances away from the interfaces. The problem of an infinite central active piezoelectric transformer is formulated by means of integral transforms and reduced to the solution of a system of Fredholm integral equations of the second kind. Numerical calculations are carried out, and the fracture mechanics parameters such as stress intensity factor, energy release rate and energy density factor based on both permeable and impermeable crack models are shown graphically and compared.

### **Chapter 5 Infinite tensile piezoelectric cylinder with a flat annular crack**

We investigate the electroelastic response of a flat annular crack in a piezoelectric cylinder of finite radius under model I loading. By the use of Fourier and Hankel transforms, the mixed boundary value problem is reduced to a singular integral equation. Numerical values on the stress intensity factor, energy release rate and energy density factor for piezoelectric cylinders are obtained to show the influence of applied electric fields, and the results are presented in graphical form. Special cases of a penny-shaped crack and a circumferential edge crack are also discussed.

### **Chapter 6 Electroelastic analysis of a piezoelectric cylindrical fiber with a penny-shaped crack embedded in a matrix**

The electroelastic response of a penny-shaped crack in a piezoelectric cylindrical fiber embedded in an

elastic matrix is investigated. Fourier and Hankel transforms are used to reduce the problem to the solution of a pair of dual integral equations. They are then reduced to a Fredholm integral equation of the second kind. Numerical values on the stress intensity factor, energy release rate and energy density factor for piezoelectric composites are obtained to show the influence of applied electric fields.

## **Chapter 7 Electroelastic response of a flat annular crack in a piezoelectric fiber surrounded by an elastic medium**

Following the theory of linear piezoelectricity, we consider the electroelastic problem of a flat annular crack in a piezoelectric fiber embedded in an elastic medium. Fourier and Hankel transform techniques are employed to formulate the mixed boundary value problem as a singular integral equation. The stress intensity factor, energy release rate and energy density factor are computed for some piezoelectric composites, and the influence of applied electric fields on the normalized values is displayed graphically.

## **Chapter 8 Impact response of a piezoelectric ceramic cylinder with a penny-shaped crack**

The dynamic electroelastic response of a penny-shaped crack in a piezoelectric ceramic cylinder under normal impact is investigated. A plane step pulse strikes the crack and stress wave diffraction takes place. Laplace and Hankel transforms are employed to reduce the transient problem to the solution of a pair of dual integral equations in the Laplace transform plane. The solution of the dual integral equations is then expressed in terms of a Fredholm integral equation of the second kind. A numerical Laplace inversion technique is used to compute the values of the dynamic stress intensity factor, energy release rate and energy density factor for some piezoelectric ceramics, and the results are graphed to display the electroelastic interactions.

## **Chapter 9 Electroelastic intensification and domain switching near plane strain crack in rectangular piezoelectric material**

The effects of crack face boundary conditions and localized polarization switching on the piezoelectric fracture have been discussed. This chapter consists of two parts. In the first part, the electroelastic problem of an infinite piezoelectric material with a crack was formulated by means of integral transforms and the solution was solved exactly. The electroelastic fields were expressed in closed form and the fracture mechanics parameters such as energy release rate were obtained for the permeable, impermeable and open crack models. In the second part, finite element analysis was carried out to study the crack behavior in a rectangular piezoelectric material by introducing a model for polarization switching in local areas of electroelastic field concentrations. A nonlinear behavior induced by localized polarization switching was observed between the fracture mechanics parameters and applied electric field.

## **Chapter 10 Conclusions**

The main results and conclusions of the present research work are summarized.

# 論文審査結果の要旨

本論文は、電子・電気機械デバイスの設計・開発および信頼性・耐久性評価のための圧電材料システムの電気破壊力学的挙動に関する理論的研究成果をまとめたもので、全編10章からなる。

第1章の序論では、本研究で対象とした圧電材料システムの特徴・応用および電気弾性破壊力学に関する研究の位置付けを述べると共に、本研究の目的と意義を明らかにしている。

第2章～第4章は、引張りを受ける2次元き裂を有する圧電材料システムを対象に、平面ひずみ電気弾性解析を行ったもので、電気破壊力学に関する基礎的挙動を解明している。圧電分極方向はき裂面に垂直である場合を考えた。まず、第2章では、縁に垂直なき裂を有する圧電セラミックス帯板を取り上げ、電気弾性・有限要素解析を行い、応力拡大係数・エネルギー解放率・エネルギー密度等の破壊力学パラメータに及ぼす電場・境界の影響を明らかにしている。また、き裂面電氣的境界条件についても考察し、き裂に垂直方向の電束密度をき裂面上で零と仮定した近似境界条件の矛盾点を指摘している。第3章では、圧電層と弾性体からなる界面に垂直なき裂を有する圧電積層材料の、第4章では、界面に垂直なき裂を有する中央駆動圧電トランスの電気弾性解析を行い、き裂の挙動に及ぼす電場・材料特性・界面の影響を解明している。

第5～8章では、引張りを受ける圧電材料システムを対象に、分極方向に垂直な3次元き裂の電気弾性挙動を明らかにしており、まず、第5章では、円環状き裂を有する圧電円柱の、また、第6章および第7章では、円板状および円環状き裂を有する圧電繊維複合材の軸対称電気弾性問題を取り上げ、電気破壊力学パラメータを解明している。続く、第8章では、円板状き裂を有する圧電円柱の電気弾性衝撃応答問題を取り上げ、応力・電場の動的特異挙動を解明している。

第9章は、き裂を有する圧電セラミックスの平面ひずみ問題を取り上げ、電気弾性・非弾性解析を行ったもので、き裂の挙動について総合的検討を加えている。また、き裂近傍における局所分域回転を考慮した有限要素解析法を開発して、圧電セラミックス帯板の非線形破壊力学的挙動解明に成功している。

最後に、第10章の結論では、各章で述べた内容を概括すると共に、得られた知見を整理して本論文の統括としている。

以上要するに、本研究は、圧電材料システムの電気破壊力学的挙動の理論的解明に成功し、高性能な電子・電気機械デバイスの設計・開発・評価に資する結果を提供したもので、材料加工プロセス学の発展に寄与するところが少なくない。

よって、本論文は博士(工学)の学位論文として合格と認める。